Design approach for diaphragm action of roof decks in precast concrete building under earthquake

L. Ferrara & G. Toniolo
Department of Structural Engineering, Politecnico di Milano, Milan, Italy

ABSTRACT: The response of a structure to earthquake relies on the diaphragmatic behaviour of slabs at both storey and roof level. Cast-in-situ r/c slabs can easily behave as rigid diaphragm. The same holds for slabs built with precast elements and completed through in situ casting, provided peripheral ties and suitable reinforcement at connections with vertical elements is provided. Long span roofing of precast r/c industrial buildings generally consist of “panel elements” connected through mechanical devices, without any cast-in-situ completion, or even interposed with skylights. Diaphragm action can be still relied upon if the in plane rotational equilibrium of elements can be guaranteed by forces at peripheral connections: for this case a simplified design approach is proposed and validated in this paper.

1 INTRODUCTION

Seismic design of building structures is aimed at providing lateral-force resisting systems with sufficient strength, stiffness and ductility to ensure a safe behaviour during earthquakes. In the whole procedure a key role is played by the diaphragm behaviour of floor and roof systems, which transfer seismic forces at each level in a uniform way to the vertical elements of the resisting system. Poor building behaviours and even collapses due to improperly designed and detailed diaphragms and connections were reported in several earthquakes during last decades. For example, the collapse of several parking structures during Northridge earthquake (1994) was rightly attributed to a neglected/underestimated diaphragm flexibility. This deficiency caused drift demands for which the elements of the gravity load resisting system were not designed and which increased second order effects of gravity loads (Fleischmann et al., 1998; Wood et al., 2000; Farrow and Fleischmann, 2003; Fleischmann and Farrow, 2004).

The in-plane stiffness of floor and roof systems generates structure redundancy towards earthquake actions, and consequently provides restraint of uncoupled equilibrium deformations of vertical subsystems. This gives rise to compatibility forces in the diaphragm, besides equilibrium forces. The magnitude of compatibility forces depends on the in-plane distribution and orientation of vertical subsystems and on their relative stiffness. In order to properly and effectively generate diaphragm action, the required stiffness of horizontal subsystems must be accompanied by sufficient strength to withstand the resulting compatibility forces.

Two main issues have hence to be considered in the seismic design of diaphragms: the proper evaluation of in-plane seismic forces, namely their values and distribution along the height of the building, and how these forces are transmitted to vertical elements, which resist the lateral actions.

The present study is focused on precast one storey buildings, as widely used for industrial and commercial halls.

Floor and roof systems normally employed in precast structures consist of an assembly of panels such as double-tee units, connected to each other by point connections along the slab edges. Since these elements possess a large in-plane stiffness, seismic design of precast diaphragms is essentially a connection design problem (Clough, 1982; Davies et al., 1990). Also special roof elements are employed, made of thin walled folded plates of different shapes, normally 2.5 m wide and up to 20–25 m long, laid in alternatively with skylights (Fig. 1) These elements are connected to the beams through pins or other suitable mechanical devices, which, besides preventing from the loss of seating during earthquake induced, have to be designed to transmit in-plane diaphragm forces, according to a suitable design model as illustrated in forthcoming
2 DIAPHRAGM MODEL

In order to show the basic principles of the model proposed to evaluate diaphragm forces on connections, reference is made to a sample building structure consisting of \( p \) bays transverse to the direction of the applied earthquake action. The lateral load resisting system hence consists of \( p + 1 \) frames with the same lateral stiffness, assuming that all the columns have the same cross sections.

Denoting by \( F_h \) the total inertia force due to earthquake, a force equal to \( F' = F_h/(p + 1) \) would pertain to each frame, if a perfect diaphragm behavior of the roof deck is assumed. On the other hand, if a complete lack of diaphragm action is assumed, also taking into account the statically determinate arrangement of the structure, lateral frames would take a force equal to \( F''_{\text{lat}} = F_h/2p \) whereas each internal frame would take a force equal to \( F''_{\text{int}} = F_h/p \).

The effect of diaphragm action can be hence quantified as follows, respectively for lateral and internal frames (Fig. 2):

\[
\Delta F_{\text{lat}} = F' - F''_{\text{lat}} = \frac{F_h}{p+1} - \frac{F_h (p-1)}{2p} = \frac{F_h (p+1)}{2p (p+1)}
\]
\[
\Delta F_{\text{int}} = F' - F''_{\text{int}} = \frac{F_h}{p+1} - \frac{F_h}{p} = \frac{F_h}{p(p+1)}
\]

In the case of two bays \( (p = 2) \), denoting as \( F = F_h/4 \) the inertia force of half a bay, with reference to Fig. 2 the transfer force can be evaluated as:

\[
0.5 \Delta F_{\text{int}} = -\Delta F_{\text{lat}} = F_h/12 = F/3 = \Delta F
\]

2.1 Interconnected double-tee elements

In this simple case the diaphragm force \( Q \) can be applied at mid-span, considering the skew-symmetry of the situation, as shown in Fig. 2(b).

For each of the \( n \) double tee elements of the roof of a single bay, a share of the diaphragm force acts

\[
Q = \frac{\Delta F}{n}
\]

which shall be added or subtracted to the average force \( F_0 = F/n \).

For the equilibrium of the element, the forces at the supports are equal to:

\[
R = F_0/2 \pm Q/2
\]

where the forces along the slab edges are:

\[
S = Ql/mb
\]

where \( m \) is the number of connections along one edge.

Connections shall be hence designed to withstand the above calculated forces.

The end elements have a free lateral edge (Fig. 2d): in this case the equilibrium is guaranteed by the two forces \( H_1 \) and \( H_2 \) as indicated in the same figure:

\[
H_1 = Q \frac{1}{2} \frac{1 - d_2/b}{d_2 - d_1}
\]
\[
H_2 = Q \frac{1}{2} \frac{1 - d_1/b}{d_2 - d_1}
\]
The model herein presented guarantees the equilibrium and not the compatibility of deformations and its use can be justified for an ultimate limit state check, provided a sufficient ductility of connections is ensured.

2.2 Spaced roof elements

For precast roof elements placed with interposed skylights, a certain degree of diaphragm action can be still guaranteed if the connections with beams are able to provide a restraint versus in-plane rotations. In Figure 3b a scheme of half an element is shown, with its share of the diaphragm force $Q = \Delta F/n$. At the connections with the beam the following forces arise:

\[ R = F/2 \pm Q/2 \]
\[ H = Ql/2b \]

3 DESIGN EXAMPLE

The approach proposed above has been hereafter applied to a “real” design case. Reference has been made to a sample building 50 m long and 30 m wide. The structure, as schematically shown in Fig. 4, is arranged on a 10 x 15 m grid, with 10 m span beams and 15 m long roof elements, spaced 1.5 m to each other. Columns have a square cross section 500 x 500 mm$^2$ and are 5.5 m high.

The weight of the roof elements has been set equal to 3.2 kN/m$^2$ (including the share of the snow load), plus the weight of beams (5.0 kN/m). A share of the cladding panel weight, equal to 8.0 kN/m, has been also considered in the seismic weight, whereas the effects of claddings on the structure stiffness has been, as commonly done, neglected.

The total seismic weight is hence equal to:

- roof deck: \(30 \times 5.0 \times 3.2 = 4800 \text{ kN}\)
- beams: \(3 \times 50 \times 5 = 750 \text{ kN}\)
- cladding panels: \(2 \times (30+50) \times 8 = 1280 \text{ kN}\)

\[
\text{total seismic weight } W = 6830 \text{ kN}
\]

to which a vibrating mass \(m = W/g = 696000 \text{ kg}\) corresponds.

The lateral stiffness is evaluated considering the cracked cross section stiffness of columns, which is assumed, for the sake of simplicity, to half the gross cross section stiffness:

\[
I_{\text{cracked}} = \frac{1}{2} \frac{bh^3}{12} = \frac{1}{2} \frac{500}{12} \text{ mm}^4 = 2604.2 \times 10^4 \text{ mm}^4
\]

and, assuming \(E = 30 \text{ kN/mm}^2\)

\[
k_\delta = \frac{3EI_{\text{cracked}}}{h^3} \approx 1530 \text{ kN/m}
\]

The vibration period of the structure can be hence calculated as:

\[
T_1 = 2\pi \sqrt{\frac{m}{\sum k_\delta}} = \sqrt{\frac{696000 \text{ kNsec}^2/m}{18 \times 1530 \text{ kN/m}}} = 1.0 \text{ sec}
\]

where the summation of lateral stiffness \(k_\delta\) has been extended to the total number (18) of columns of the structure. For this period the value of the elastic
The force on each half bay is equal to

\[ F = F_0/4 = 199.25 \text{ kN} \]

and the transfer force is hence equal to

\[ \Delta F = F_0/12 = 66.4 \text{ kN} \]

### 3.1 Interconnected double-tee elements

Assuming equal to 20 the number of interconnected roof elements for each bay, the diaphragm force pertaining to each of them can be calculated:

\[ F_0 = F/n = 199.25/20 \text{ kN} = 9.96 \text{ kN} \]

\[ Q = \Delta F/n = 66.4/20 \text{ kN} = 3.32 \text{ kN} \]

The forces on the edge connections are:

\[ R = F_0/2 + Q/2 = 9.96/2 + 3.32/2 = 6.64 \text{ kN} \]

along the lateral beams, and

\[ R = F_0/2 - Q/2 = 9.96/2 - 3.32/2 = 3.32 \text{ kN} \]

along the central one.

Assuming that 5 connection devices are placed along each edge, the forces S can be evaluated:

\[ S = S = Ql/mb = \frac{3.32 \times 15}{5 \times 2.5} = 3.98 \text{ kN} \]

At edge supports of the end elements, for \( d_1 = 0.5 \text{ m} \) and \( d_2 = 2.0 \text{ m} \), the following forces arise:

\[ H_1 = \frac{Q}{2} \frac{1 - d_1/2}{d_2 - d_1} = 3.32 \frac{151 - 2/2.5}{2} = 3.32 \text{ kN} \]

\[ H_2 = \frac{Q}{2} \frac{1-d_1/2}{d_2 - d_1} = 3.32 \frac{151 - 0.5/2.5}{2} = 13.28 \text{ kN} \]

### Table 1. Results for interconnected roof elements.

<table>
<thead>
<tr>
<th></th>
<th>Symmetric mass</th>
<th>Non-symmetric mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral</td>
<td>R 1.10</td>
<td>1.04</td>
</tr>
<tr>
<td></td>
<td>S 0.60</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>H_1 0.03</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>H_2 0.01</td>
<td>0.45</td>
</tr>
<tr>
<td>Central</td>
<td>R 1.12</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>S 0.60</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>H_1 0.76</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>H_2 0.32</td>
<td>0.25</td>
</tr>
</tbody>
</table>

### 3.2 Spaced roof elements

Assuming equal to 15 the number of spaced roof elements for each bay, the diaphragm force pertaining to each of them can be calculated:

\[ F_0 = F/n = 199.25/15 \text{ kN} = 13.28 \text{ kN} \]

\[ Q = \Delta F/n = 66.4/15 \text{ kN} = 4.43 \text{ kN} \]

The forces on connections turn out to be equal to:

\[ R = F_0/2 + Q/2 = 6.74 + 2.22 = 8.96 \text{ kN} \]

\[ H = Ql/2b = 4.43 \times 15/(2 \times 1.5) = 22.15 \text{ kN} \]

along the lateral beam, and

\[ R = F_0/2 - Q/2 = 6.74 - 2.22 = 4.56 \text{ kN} \]

\[ H = 22.15 \text{ kN} \]

along the central beam.

The values calculated through the simplified model will be checked by a more rigorous structural analysis as hereafter described.

### 4 MODAL ANALYSIS

A check has been made through a modal analysis performed on a 3D model of the sample structure, employing the code Cast3m. Both arrangements of
Figure 5a. Vibration modes – interconnected roof elements with symmetric mass distribution.

Mode x: $T = 1.02$ sec, $p = 99.98\%$

Mode y: $T = 1.01$ sec, $p = 99.97\%$

Mode z: $T = 0.72$ sec

Figure 5b. Vibration modes – interconnected roof elements with non-symmetric mass distribution.

Mode x: $T = 1.03$ sec, $p = 97.52\%$

eccentricity y

Mode y: $T = 1.01$ sec, $p = 99.30\%$

eccentricity x

Mode z: $T = 0.71$ sec – eccentricity y

In Table 1 the ratios between the forces given by modal analysis and those calculated according to the proposed simplified approach are listed. It has to be remarked that for non symmetric mass distribution,
the values of forces calculated through the simplified design approach have been multiplied by 1.2.

5 CONCLUDING REMARKS

It has been shown in this study, through a modal analysis performed on a sample structure with different arrangements of the roof deck, that the proposed design method yields reliable results as far as the evaluation of transfer and diaphragm forces is referred to the interconnected roof elements (Table 1). Less reliable predictions have been obtained with reference to spaced roof elements, mainly for non symmetric mass distribution. In this case a precise structural analysis applied to a 3D model seems to be necessary.

REFERENCES