Direct load transmission in sandwich slabs with lightweight concrete core

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ABSTRACT: This paper presents a variational energy based model to predict the cracking and ultimate load of hybrid FRP-concrete sandwich bridge slabs under direct load transmission in the support region. The slab consists of three layers: a glass fiber-reinforced polymer composite (GFRP) sheet with T-upstands for the bottom skin, lightweight concrete (LC) for the core and a thin layer of ultra high performance reinforced concrete (UHPFRC) as top skin. Different LC types were used: a low and a high density sand lightweight aggregate concrete (SLWAC) and an all-lightweight aggregate concrete (ALWAC). A bottle-shaped compressive strut and a continuous transverse tensile tie allowed for stress redistribution after cracking due to LC softening. Experimental cracking and ultimate loads of deep short span beams could be accurately modeled in this way. The arch rise of the compressive strut decreased significantly after concrete cracking.

1 INTRODUCTION

A novel concept for a hybrid GFRP-concrete sandwich bridge deck system was proposed by the authors (Keller et al. 2007, Schaumann et al., 2008). The sandwich slab consists of three layers: a GFRP element for the tension skin, which also serves as formwork, lightweight concrete as core material and ultra high performance fiber-reinforced concrete for the compression skin. No additional shear reinforcements of the core (e.g. rebar, studs) is considered in order to ensure a simple and cost-effective manufacturing procedure for the slab.

The shear behavior of the proposed hybrid sandwich system was experimentally investigated for two different types of LC: sand lightweight aggregate concrete and all lightweight aggregate concrete (acc. to Faust 2002) and two different shear span-to-depth ratios \( (a/d = 8 \text{ and } 1.6) \), where \( a \) is the distance from the load to the support axis and \( d \) the effective depth. The ratio \( a/d = 8 \) corresponds to a shear failure in the field (beam mechanism), while for \( a/d = 1.6 \) the load is directly transmitted through a compressive strut to the support (direct load transmission through a compression diagonal). It has been experimentally and numerically verified that the compression diagonal, rather than being parallel-sided, is bowed or so-called bottle-shaped between the loading point and the support, and consequently tension transverse to the diagonal results (ACI-445 1998; Brown 2005; Foster 1998; Schlaich et al. 1987). The stress flow in the concrete is often idealized as a truss consisting of compressive struts and ties necessary for the equilibrium.

A fracture mechanics-based model to predict the shear resistance of the unreinforced LC core for high shear-span ratios (beam mechanism) was established, which showed that not only static strength data but also fracture mechanics properties must be considered to accurately describe shear behavior. The present paper proposes a model for the direct load transmission behavior, which takes the softening of the LC core after cracking into account.

2 EXPERIMENTAL INVESTIGATION

2.1 Specimens

For the conducted direct load transmission experiments, 1200 mm long, 400 mm wide and 200 mm deep specimens were prepared (see Figs 1 and 2). A detailed description of the experiments is provided in Schaumann et al. (2008).

The top skin consisted of a 30 mm normal concrete (NC) layer, since beam design showed that it was not necessary to use UHPFRC for the experiments. A standard pultruded GFRP sheet with T-upstands (Plank element 40HDx500 from Fiberline, (2006)) was used for the bottom skin. Three different LCs were used as core material: two SLWAC mixtures of an average density of 900 and 1300 kg/m² and one ALWAC mixture of 1000 kg/m². Two different FRP-LC interfaces were investigated: pure mechanical interlocking between
LC and FRP T-upstands and the adhesive bonding of the interface. Within the scope of this investigation, from 8 specimens tested only the specimen configurations that exhibited full composite action in the FRP-LC interface were considered. Table 1 gives an overview of the six specimen configurations used and their labeling.

2.2 Materials

The GFRP Plank element exhibited a tensile strength of 240 MPa and a Young’s modulus of 23 GPa, according to Fiberline (2006). A cold-curing two-component epoxy adhesive was used for the FRP-LC interface (SikaDur 330 from Sika, axial tensile strength of 38 MPa). A standard mixture of self-compacting concrete was used for the NC-layer (mean compressive strength of 51.2 MPa, Young’s modulus of 29.7 GPa). The SLWAC mixtures (LC900 and LC1300) consisted of expanded clay aggregates (Liapor F3, Ø4–8 mm), normal sand, cement and water. The LC1000 ALWAC was composed of the same expanded clay aggregates and expanded glass aggregates (Liaver, Ø1–2 and 2–4 mm), cement, filler, adjuvants and water. Mean compressive strengths, $f_{lc,m}$, and Young’s Moduli, $E_{lc,m}$, were determined according to Swiss Code SIA 162/1, as well as mean splitting tensile strengths, $f_{lctsp,m}$, according to Swiss Code SIA 262 on three cylinders (Ø = 160 mm) for each mixture. The LC densities were measured on cylinders after storage in a climate room at 20°C and 95% humidity for 28 days.

According to Faust (2002), characteristic lengths, $l_{ch}$, for the SLWAC compositions were estimated as 150 mm, whereas 40 mm was assumed for the ALWAC mixture. The material properties are summarized in Table 2.

2.3 Experimental setup

All specimens were simply supported on rollers with a span of 600 mm and subjected to three-point bending using a displacement-controlled hydraulic jack at mid-span, as shown in Figure 2. The ratio $a/d$ was 300/185 = 1.6 and the angle of the compression diagonal $\alpha = 33.7^\circ$.

Omega-shaped extensometers enabled deformations over a gage length of 100 mm on the lateral concrete surfaces to be measured. On one specimen side, 2 × 5 extensometers (labeled O20–O29) transverse to the diagonals measured deformations (due to tension) at distances of 40 mm along each diagonal. On the other specimen side, 5 extensometers (O34–O38) were fixed parallel to one diagonal at distances of 30 mm to measure the longitudinal deformation distribution (due to compression) transverse to the diagonal, as shown in Figure 2.

2.4 Main experimental results

All specimens showed full composite action and almost linear elastic behavior up to ultimate load.
Table 3. LC cracking loads, specimen ultimate loads and arch rises at cracking and ultimate loads.

<table>
<thead>
<tr>
<th>Beam</th>
<th>$F_{cr}$ [kN]</th>
<th>$F_u$ [kN]</th>
<th>$y_{0,cr}$ [mm]</th>
<th>$y_{0,u}$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>900E-1/900E-2</td>
<td>60/65</td>
<td>98/83</td>
<td>60/57</td>
<td>40/42</td>
</tr>
<tr>
<td>1300E-1/1300E-2</td>
<td>87/80</td>
<td>204/185</td>
<td>73/77</td>
<td>40/42</td>
</tr>
<tr>
<td>1000/1000E</td>
<td>45/50</td>
<td>164/201</td>
<td>90/85</td>
<td>29/27</td>
</tr>
</tbody>
</table>

Shear failure occurred in the LC core at the ultimate loads shown in Table 3, which were not correlated to the LC compressive strength. Figure 3 shows representative load-deformation responses of extensometer O22 (located in the middle of one diagonal).

From these curves and the corresponding ones of O27 on the opposite diagonal, the LC cracking load, $F_{cr}$, was determined, as summarized in Table 3. Crack initiation was assumed when a sudden increase in the deformations occurred. The SLWAC specimens exhibited significantly higher cracking loads (60 to 87 kN) than the ALWAC specimens (45 and 50 kN).

Typical transverse deformation distributions (due to tension) measured along the diagonal of 1000E are shown in Figure 4 in 10 kN load steps. The measurements indicated a parabolic distribution of deformations with maximum deformations measured in the middle of the diagonal. LC cracking occurred at load step 50 kN, corresponding to deformations of 0.030 mm. When cracking loads were exceeded, the measured deformations increased rapidly.

A representative development of the deformation distribution (due to compression) transverse to the diagonal at different load steps, up to ultimate load, is illustrated in Figure 5 for specimen 1300E-2. Up to the load step 75 kN, deformations were almost evenly distributed over a large width (with a maximum value of $-0.03$ mm). Subsequently, after cracking at 80 kN, deformations started to increase significantly in a much narrower range of approximately 45 mm on each side of the diagonal (in O36–O38). At ultimate load, deformations reached values of $-0.90$ mm.

3 MODELING

3.1 Softening of lightweight concrete

The two types of lightweight concrete (SLWAC and ALWAC) showed significant differences in material properties and structural performance. The cracking load of the ALWAC specimens was significantly lower than that of the SLWAC.

To take the influence of concrete brittleness into account, the post-peak softening behavior is considered. The fracture energy, $G_f$, is determined through
Table 4. LC fracture mechanics characteristics.

<table>
<thead>
<tr>
<th>Lightweight concrete</th>
<th>(l_{ch}) [mm]</th>
<th>(G_f) [N/m]</th>
<th>(w_{crit}) [(\mu m)]</th>
<th>(r) [1/mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>LC900 (SLWAC)</td>
<td>150</td>
<td>14.7</td>
<td>86</td>
<td>38</td>
</tr>
<tr>
<td>LC1300 (SLWAC)</td>
<td>150</td>
<td>23.6</td>
<td>69</td>
<td>48</td>
</tr>
<tr>
<td>LC1000 (ALWAC)</td>
<td>40</td>
<td>10.2</td>
<td>27</td>
<td>123</td>
</tr>
</tbody>
</table>

Figure 6. (a) Development of the material softening; bilinear and exponential approximation and (b) Young’s modulus before and after cracking for LC1000.

the characteristic length, \(l_{ch}\), the uniaxial tensile strength, \(f_{ct,m}\), and the Young’s modulus, \(E_{ct,m}\) (acc. to Hillerborg, 1983). Assuming a typical bilinear degradation of the softening curve with a kink located at 0.33\(w_{crit}\) and 0.25\(w_{crit}\), the critical crack openings, \(w_{crit}\), could be determined (Faust 2002), as listed in Table 4 and shown in Figure 6a.

No tensile stresses can be transmitted through cracks of widths larger than the critical crack opening. Compared to the SLWAC, the lower fracture energy of the ALWAC resulted in a smaller critical crack opening, and thus the ALWAC rapidly lost, i.e. under small deformations, the ability to transmit tensile stresses through a crack.

The resulting critical crack openings are indicated in Figure 4 for a typical ALWAC specimen. The values correspond well to the measured deformations at LC cracking.

3.2 Continuous direct load transmission model

A major problem in the use of strut-and-tie models for direct load transmission is the definition of the effective widths on which the compression and tensile forces act. The proposed values were always fitted on experimental results. Furthermore, the systems are statically determined and, therefore, do not allow load or stress redistribution. In this respect, statically indeterminate systems could be more advantageous, since they allow stress redistribution resulting from nonlinear material behavior after concrete cracking (LC softening) to be taken into account.

In the following, an extension of the truss model with its discrete system of ties to a system based on a continuum, with an infinite number of ties, is proposed. The model consists of a bottle-shaped strut, which transmits the diagonal compressive force and, to fulfill equilibrium, generates tension transverse to the strut in the LC (represented as LC ties), as illustrated in Figures 7 and 8. The initially undeformed shape of the compressive force flow is geometrically defined by Equation 1:

\[
y(x) = y_0 \cdot \sin^2 \left( \frac{x}{L} \right) + y_c
\]  

where \(y_0\) is the rise of the bottle-shaped arch at \(L/2\), \(L\) is the span of the bottle-shaped arch, defined by the total length of the diagonal, \(L_{tot}\), minus the length of the nodal zones, and \(y_c\) is the offset of the bowed compressive strut from the diagonal, which corresponds to a quarter of the width of the load introduction surface at the nodes. The \(\sin^2\)-term is chosen as it well represents typical bottle-shaped force flows. The undeformed shape exhibits a maximum rise \(y_0 + y_c\) at \(x = 0.5L\), and inflection points at \(x = 0.25L\) and 0.75\(L\). Based on the experimental results and Foster (1998) and Brown (2005), the arch rise at concrete cracking, \(y_{0,cr}\), is assumed to be higher than the arch rise at ultimate load, \(y_{0,u}\).

The analysis of the model is based on a variational energy principle, where variations of the inner and outer potential energies of the system are equal at the
Young's modulus of the LC concrete, $E_y$, of the ties. Axial deformations of the struts do not influence significantly the results and are not taken into consideration.

The external load, $C$, representing the diagonal compressive force, induces an axial displacement, $u$, which deforms the undeformed bowed strut shape, $y(x)$, to the deformed shape with the deformation $w(x)$, relative to $y(x)$. A virtual displacement $\delta u$ is applied, which deforms the bowed strut to achieve a new equilibrium state, $w(x) + \delta w(x)$.

Equilibrium is achieved when, for any virtual deformation state, the sum of the variations of the inner and outer potential energies is zero, expressed as follows:

$$\delta \pi_i + \delta \pi_o = 0$$

where $\delta \pi_i$ = the variation of the inner potential energy and $\delta \pi_o$ = the variation of the outer potential energy, that is the potential energy of the external loads:

$$\delta \pi_o = C \cdot \delta u$$

The virtual displacement $\delta u$ is the integration of the reduction of the projected length of the strut, while the variation of $\delta \pi_i$ includes contributions from the elongations of the ties and shear deformations resulting from different elongations of two adjacent ties:

$$\delta \pi_i = \int \sigma_y \cdot \delta \varepsilon_y \, dV + \int \tau_{xy} \cdot \delta \gamma_{xy} \, dV$$

where $V$ is the volume enclosed by the bottle-shaped strut, $\sigma_y$ and $\tau_{xy}$ are the axial and shear stresses of the ties, and $\delta \varepsilon_y$ and $\delta \gamma_{xy}$ are the virtual strains and distortions resulting from the virtual displacement.

The axial stresses, $\sigma_y$, are calculated through the Young’s modulus of the LC concrete, $E_y$, and the axial strain of the tie, $\varepsilon_y$, resulting from the elongation, $w(x)$, related to its original length, $y(x)$.

When the tensile strength of the LC is exceeded, material softening occurs. To further develop the proposed model, the bilinear post-peak strength decrease is approximated by an exponential function, as shown in Figure 6a for LC1000.

To take the softening behavior in the model into account, a similar exponential decrease is assumed for $E_y$, as obtained for the post-peak tensile strength. Thus, $E_y$ can be expressed as a two-stage function, as shown in Figure 6b, and described by Equation 5:

$$E_y(w(x)) = \begin{cases} 
1 & \text{for } w(x) < w_{lc}(x) \\
e^{-r(w(x) - w_{lc}(x))} & \text{for } w(x) \geq w_{lc}(x)
\end{cases}$$

where $r$ is an exponential factor (given in Table 4), and $w_{lc}(x) = f_{lc,m}/E_{lc,m} \cdot y(x)$ is the deformation when the uniaxial LC tensile strength is reached, as illustrated in Figure 9a.

The shear stresses, $\tau_{xy}$, are calculated from the LC shear modulus, $G_{xy}$, and the distortion, $\gamma_{xy}$, corresponding to the difference in elongations of two adjacent ties $dw(x)/dx = w'(x)$. It is assumed that the LC shear modulus exhibits a similar decrease to the Young’s modulus. The initial values, $G_{lc,m}$, can be calculated from $G_{lc,m} = E_{lc,m}/(1 + \nu)$ assuming a Poisson’s ratio of $\nu = 0.2$ (Faust, 2002); the resulting values are listed in Table 2. By combining Equations 3–5, Equation 2 is only expressed by material properties and $w(x)$ and $\delta w(x)$. The non-linear equation is solved iteratively, assuming an arch rise, $y_0$, and increasing the external load in small steps. At first, as long as the deformed shape, $w(x)$, is below $w_{lc}(x)$, the relationship between stresses and deformations remains linear (indicated with a square in Figs 6 and 9 for LC1000). Subsequently, $w(x) = w_{lc}(x)$ is reached (indicated with a star symbol) and when $w(x) > w_{lc}(x)$, $E_y$ and $G_{xy}$ start to decrease (indicated with a circle) and deformations rise exponentially. The stiffness of the system then progressively decreases because the extension of the zone exceeding $w_{lc}(x)$ increases. In the range of $x = 0–0.25L$, deformations become negative due to the negative second derivative of the initial shape ($y'' < 0$), see Figure 9a. This, however,
is compatible with the hydrostatic compressive stress state existing in nodal zones of concrete structures.

Depending on the assumption of the arch rise, the cracking load, \( C_{cr} \), or ultimate load, \( C_u \), of the strut can be determined and is reached when the algorithm diverges, that is, when corresponding deformations become infinite. The cracking load of the strut is then transformed to the cracking load of the specimen by \( F_{cr} = 2 \cdot C_{cr} \cdot \sin \alpha \). For practical reasons, calculations were stopped at a deformation of 0.2 mm in the middle of the strut, \( \nu(L/2) \).

In order to apply the proposed model, the arch rises at LC concrete cracking and at ultimate load had to be determined first through calibration to the experimental results. The arch rise was varied from 10 to 100 mm and the corresponding loads, \( F \), at 0.2 mm deformation were calculated for the three LC types. From these values and the measured cracking and ultimate loads, the arch rises at cracking loads, \( y_{0,cr} \), and ultimate loads, \( y_{0,u} \), were determined. As assumed, the arch rises at cracking were much higher than at ultimate load, see Table 3.

4 RESULTS AND DISCUSSION

4.1 Continuous direct load transmission model

The proposed model for the direct load transmission behavior is an extension of a typical statically determinate strut-and-tie model to a model consisting of a bottle-shaped strut with an infinite number of ties. The strut transmits the diagonal compressive force and, to fulfill equilibrium, generates tension in the ties. The statically indeterminate system enables the model to take into account stress redistributions that occur after concrete cracking as a result of the non-linear material behavior (LC softening). Taking the softening behavior into consideration allows accurate modeling of the differences between the SLWAC and ALWAC specimens, which exhibit different brittleness.

Although the suggested model needs to be calibrated on experimental results, that is, the arch rises of the strut at concrete cracking and ultimate load have to be deviated from measured cracking and ultimate loads, the model is able to describe the transverse deformation of the strut during concrete cracking. After cracking, the arch rise decreases that compares well with experimental observations, where the width of the compression strut was reduced by the initiating and propagating cracks parallel to the strut.

4.2 Comparison of experimental and predicted deformations at cracking

The transverse deformations (due to tension) at LC cracking are shown in Figure 3 for specimen 1000E. Good agreement with the experimental results was found. The proposed method provided deformations at LC cracking comparable to experimental results, including the transverse compression in the nodal zones, which could also be found by an extrapolation of the experimental curves. However, the zones of deformation due to transverse tension were less extended compared to the measurements.

Subsequent to cracking, experimental deformations increased significantly and measurements were influenced by the cracks in such a way that no comprehensive and conclusive distributions of deformations along and across the diagonals could be obtained at ultimate load. Therefore, predicted and measured deformations at ultimate load could not be compared.

5 CONCLUSIONS

The direct load transmission behavior of hybrid sandwich beams consisting of a bottom FRP skin and a top concrete skin was investigated experimentally. The sandwich core consists of lightweight concretes of different brittleness (SLWAC and ALWAC mixtures). Shear failure occurred in the LC core at ultimate loads which were not correlated to the compressive strength of the compression strut between load and support axis. LC cracking and ultimate loads were modeled with a novel continuous load transmission model. The following conclusions were drawn:

1) The proposed continuous direct load transmission model consisted of a bottle-shaped strut with an infinite number of transverse ties. The statically indeterminate system allowed the stress redistribution resulting from the post-peak material softening after concrete cracking. Taking the softening into account allowed accurate modeling of the differences between the load-bearing behavior of SLWAC and ALWAC specimens.

2) The arch rise of the bowed compression strut decreased after cracking due to the available width of the strut being reduced to the distance between initiating and propagating cracks parallel to the strut.

ACKNOWLEDGEMENTS

The authors wish to acknowledge the support of Fiberverline Composites A/S, Denmark, supplier of the GFRP Plank elements, Sika AG, Zurich, Switzerland, supplier of the epoxy adhesive, Liapor, Switzerland, supplier of the LC concrete, and Prebeton SA, Avenches, Switzerland, for the fabrication of the beams. This research was funded by the project New Road Construction Concept (NR2C) of the 6th European Framework Program (Grant OFES No. 03.0318).
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