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Shear design of FRC members with little or no conventional shear reinforcement

F. Minelli & G.A. Plizzari
University of Brescia, Department DICATA, Brescia, Italy

ABSTRACT: The present paper deals with some crucial design aspects of Fiber Reinforced Concrete (FRC) beams under shear loading, with or without conventional transverse reinforcement. It focuses on shear critical beams made of plain concrete or FRC. The influence of fibers on crack formation and development, failure mode, ductility and stiffness are herein investigated. A recent analytical proposal of the authors is further developed. Statements on the minimum shear reinforcement provided by steel fibers are derived and discussed. Finally, some preliminary results on the size effect issue of FRC members in shear and a design example are reported.

1 INTRODUCTION

In spite of the vast amount of experiments that have been conducted to assess the shear capacity of structural concrete members, the behavior and design of reinforced concrete (RC) elements under shear remain an area of much concern. Design codes are continually changing and generally becoming more stringent so that structures that were designed several decades ago typically do not comply with the requirements of current codes.

Several proposals for predicting the shear capacity of beams without transverse reinforcement were published years by years: usually, they are empirical formulations, designed to fit the limited set of shear test results that are most familiar to the researchers.

It remains a pressing need to establish design and analysis methods that provide realistic assessments of the strength, stiffness and ductility of structural elements subjected to shear loading.

Adding structural (high modulus) fibers to the concrete matrix demonstrated to be quite effective in reducing the brittleness associated with shear failure, as outlined by many researchers, among them Imam et al. (1995) and Khuntia et al. (1999).

In heavy precast industry, where high strength concrete (HSC) is commonly adopted, diffused fiber reinforcement could be utilized to reduce or substitute conventional transverse reinforcement, with advantages in the production process and in the reduction of labor costs in placing and handling of rebars (Meda et al., 2005).

The present paper deals with some crucial design aspects of Fiber Reinforced Concrete (FRC) beams under shear loading, with or without conventional transverse reinforcement.

A recent analytical proposal developed in the Ph.D. thesis of Minelli (2005) and included in the FRC Italian Guidelines CNR-DT 204/2006 (2006) is further discussed and compared against more than 60 experiments available in literature. Special emphasis is devoted to the actual resistance mechanism provided by fibers and, moreover, in the way is put into design guidelines.

Moreover, shear experiments on deep beams and an analytical formulation for the design of the minimum toughness of a FRC to satisfy the minimum shear reinforcement requirement are also given.

Final goal of the ongoing research at the University of Brescia is to provide a quite simple procedure which could allow engineers to become familiar with FRC and to incorporate them in the design formulations of building codes.

2 STRUCTURAL FIBERS: AN ALTERNATIVE FOR SHEAR REINFORCEMENT

A shear analytical model (Minelli, 2005) was developed based on a quite simple adaptation of the current formulation included in the current Eurocode 2 (EC2, 2003) for shear in members without conventional transverse reinforcement. This formulation should be considered as a basis for including the effect of fibers, as the model is simple, well known, generally accepted and utilized for design purposes.

Minelli et al. (2005) outlined that the addition of a small amount of steel fibers (volume fraction V_f of 0.4–0.6%) significantly improves the structural behavior of members without transverse reinforcement: if FRC is tough enough, the collapse mechanism can alter from shear (“block mechanism”) to flexure, with a considerable increase in load bearing capacity and ductility. A brief summary of some interesting experimental results follows.

Among the 47 shear tests carried out at the University of Brescia since 2001 (most of them can be found in Minelli, 2005), the following discussion will focus on those experiments conducted to assess the influence of the minimum shear reinforcement, represented either by classical transverse reinforcement or steel fibers, and the size effect in shear.

Two series of specimens are presented herein: the first refers to beams having a total depth of 500 mm ("Small Size Specimens"), while the second consists of elements 1000 mm deep ("Large Size Specimens").

Concerning the first set of experiments, five shear-critical beams loaded with a three point loading system having a shear span-to-depth ratio of 2.5 (which is recognized to be the most critical in terms of shear strength; Kani, 1967) were tested. All beams had the same geometry and were tested for analyzing the effect of the addition of a randomly distributed fibrous reinforcement to concrete.

A beam depth of 500 mm was chosen, with a gross cover of 45 mm. The beam spanned 2280 mm, while the overall length of the specimen was 2400 mm. Two deformed longitudinal bars, having a diameter of 24 mm, were added to each specimen, corresponding to a reinforcement ratio of 1.04%.

Deeper beams (Large Size Specimens) were cast with a total depth of 1000 mm, an effective depth of 910 mm, a width of 200 mm and a span of 4550 mm (Figure 1). A three point loading scheme was chosen resulting in a span-to-depth ratio again of 2.5. The steel reinforcement ($6\phi20$ mm rebars) was located in two identical bottom layers giving a reinforcing ratio of 1.03%.

All smaller specimens were cast by using a normal strength concrete (NSC), while two different series of larger beams were tested, the first one cast in the same batch as the smaller beams, whereas the second using a high strength matrix (HSC). Table 1 reports the composition of the two different concrete batches.

One Small Specimens was cast without any transverse reinforcement (PC-50), two with the minimum amount of transverse reinforcement (MSR-50, with stirrups $2\phi8@300$ mm) as required by EC2 (2003), and two with 20 kg/m^3 of steel fibers (FRC-50) having a length of 50 mm and a diameter of 1 mm (aspect ratio $l/\phi = 50$).

The fracture properties of FRC were determined according to the Italian Standard (UNI 11039, 2003), which requires bending tests (4PBT) on small beam specimens (150 × 150 × 600 mm).

The equivalent post-cracking strengths related to the SLS and ULS were equal to $f_{eq(0-0.6)} = 2.53$ MPa and $f_{eq(0.6-3)} = 2.50$ MPa (UNI 11039, 2003), respectively.

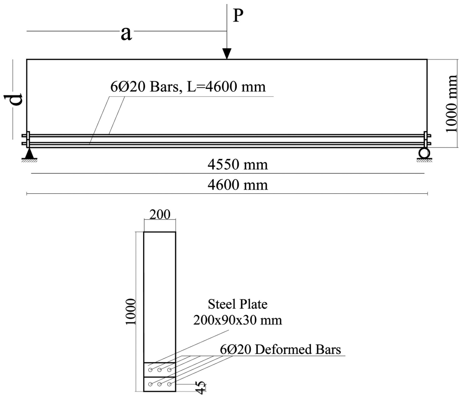

Figure 1. Geometry of Large Size Specimen.

Table 1. Mechanical properties of concrete.

	NSC	HSC
Cement Content [kg/m^3]	345	380
Maximum Aggregate Size [mm]	20	20
Plasticizer [l/m^3]	5.2	3.8
Compressive Cubic Strength [MPa]	25.7	55
Elastic Modulus [GPa]	31	37

Three Large Size Specimens were cast for each concrete strength (NSC and HSC): the reference element (PC-100), the beam containing the minimum amount of shear reinforcement (MSR-100, with stirrups $2\phi8@650$ mm) and the latter containing 20 kg/m^3 of hooked steel fibers (FRC-100). Note that the notation (1) PC refers to a beam cast with plain concrete; (2) FRC always indicates a beam with fibers as the only shear reinforcement; (3) MSR refers to a beam with minimum conventional shear reinforcement.

One should note that 20 kg/m^3 corresponds to a volume fraction very low (0,25%) that was accepted in this experimentation whose aim is the evaluation of steel fibers as a minimum transverse shear reinforcement.

Figure 2 and Figure 3 show the load-displacement curves for the large size specimens, respectively for the NSC series and HSC series.

Figure 4 exhibits the width of the main shear crack vs. the load for the HSC series. Note that the main shear crack is the average of 6 measurement performed in both shear spans (see Figure 4).

Differently form the small size specimens (Minelli et al., 2007), in larger beams the traditional shear reinforcement turned out to be significantly more effective than steel fibers, both in terms of bearing capacity and ductility. However, at service loads fibers gives better performance, compared to the MSR specimens, improving the tension stiffening effect and therefore reducing the displacement. Also cracks are fewer and

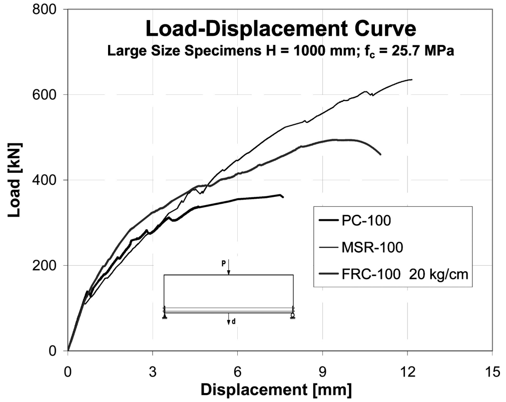

Figure 2. Load-Displacement Curve; series 1 (H = 1000 mm).

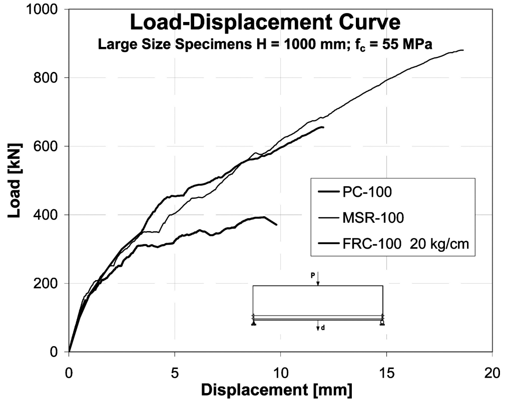

Figure 3. Load-Displacement Curve; series 2 (H = 1000 mm).

narrower (at service level) then those of MSR specimens, for both NSC and HSC series. With increasing loads and after a shear crack of around 3 mm, fibers are no longer able to resist further load and displacement, bringing the member to the "block mechanism" which characterizes the shear collapse.

As diffusely reported in the literature, the primary role of the minimum shear reinforcement is to limit the growth of inclined cracks, to improve ductility, and to ensure that the concrete contribution to shear resistance is maintained at least until yielding of shear reinforcement. In other words, it is commonly agreed that, before failure, the R/C structure must give a warning by cracking and showing a visible deflection; this represents the requirements for the minimum reinforcement ratio. Beams that do not contain web reinforcement may fail in a relatively brittle manner immediately after the formation of the first diagonal crack (Figure 4). Minimum shear reinforcement can be omitted if (1) there is no significant chance of diagonal cracking; (2) the member is of minor importance; (3) the member is part of a redundant structural system that allows substantial redistribution and, hence, will show adequate ductility.

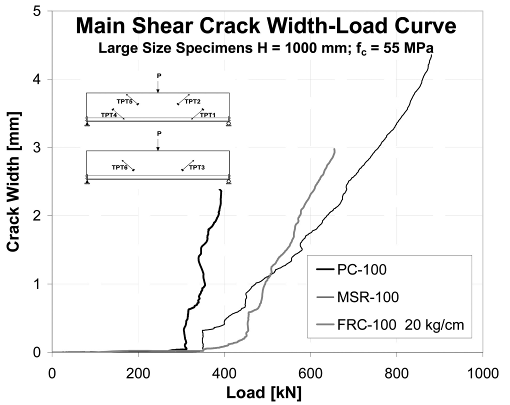

Figure 4. Shear Crack Width-Load Curve; series 2 (H = 1000 mm).

However, one should keep in mind that the shear capacity of such members can be substantially reduced by factors such as (1) repeated loading which propagates existing cracks and lowers the apparent tensile strength of concrete; (2) tensile stresses caused by restrained shrinkage strains; (3) thermal strains or creep strains; stress concentrations due to discontinuities such as web openings, (4) termination of flexural reinforcement, or (5) local deviation of tendon profiles (Collins & Mitchell, 1993).

A low amount of fibers provides all the aforementioned benefits, being a valuable and economical alternative to the traditional stirrups, whose handling and placing can be expensive especially when dealing with precast beams or, in general, with structural elements characterized by non-rectangular or square cross-sections. FRC gives the same structural response of members containing the minimum transverse reinforcement.

Table 2 reports a summary of the main results of the experimental campaign herein discussed. Note that the ultimate shear stress (v_u) as well as the ultimate shear force (V_U), were determined by considering also the self-weight of the members. In the calculation of the bearing capacity of the member, which corresponds to a flexure failure ($V_{U,FL}$), the effect of fibers (on the ultimate bending strength) was neglected.

Definitely, the addition of fiber reinforcement does not completely solve the size effect issue, especially if fibers are provided in such a low amount. On the contrary, the scale effects seem to disappear using the minimum content of transverse reinforcement. Further experimental and analytical studies, with this regard, are reported in Minelli et al. (2006).

Table 2. Summary of the main experimental values.

Specimen	P_u [kN]	v_u [MPa]	$v_u/(f_c)^{1/2}$ [−]	δ_u [mm]	$V_u/V_{u,FL}$ [−]
PC-50	216	1.22	0.24	2.74	0.52
MSR-50 1	346	1.93	0.38	9.33	0.82
MSR-50 2	302	1.69	0.33	7.03	0.72
FRC-50 1	388	2.16	0.43	10.95	0.92
FRC-50 2	308	1.72	0.34	4.77	0.73
PC-100 NSC	365	1.07	0.21	7.60	0.43
MSR-100	635	1.81	0.36	12.60	0.73
FRC-100	494	1.42	0.28	11.05	0.57
PC-100 HSC	393	1.14	0.15	9.79	0.45
MSR-100	880	2.48	0.33	18.62	0.99
FRC-100	656	1.86	0.25	12.01	0.74

3 SHEAR STRENGTH OF FRC BEAMS

As above mentioned, the model proposed by Minelli (2005) aimed at including a performance parameter of FRC in the EC2 (2003) formulation for shear. The performance parameter is conventionally assumed as the equivalent post-cracking strength defined in the Italian Standard UNI 11039 (2003). This parameter can be determined from fracture tests on small notched beams, as specified.

Since shear cracking in FRC members proved to develop in a quite stable fashion, even for crack widths of 3 mm (Figure 4), the equivalent strength related to the ultimate limit state ($f_{eq(0.6-3)}$ in UNI 11039 or f_{R4} in EN 14651 (2005) can be considered. The ability of fibers in controlling the second branch of the shear-critical crack even for big crack widths is due to their capability of bridging the two faces of a crack. By keeping cracks stable, the shear capacity of members considerably increases till, eventually, the full flexural capacity is attained.

The issue is now how to include a performance parameter for FRC. The presence of fibers all over the depth is relevant for the shear behavior, in the same way as placing longitudinal rebars along the depth of a member proved to be highly beneficial in terms of bearing capacity and overall ductility. In fact, Kuchma et al. (CEB Bulletin 237, 1997) demonstrated that, by adding three longitudinal layers of relatively low diameter bars over the depth of a specimen 1000 mm deep, the shear strength increased of about 50%, the ductility doubled and a well distributed crack pattern formed, without an early localization of any shear-critical crack.

Fibers act in providing a member with exactly the same effect; therefore, it seems reasonable to model the shear contribution of fibers as a modifier of the longitudinal reinforcement ratio (as done also by Imam et al. (1995)) throughout a factor that includes the toughness properties of fibers. The equation of EC2 for members without web reinforcement is:

$$V_{Rd,ct} = \left[\frac{0.18}{\gamma_c} \cdot k \cdot (100 \cdot \rho_1 \cdot f_{ck})^{1/3} + 0.15 \cdot \sigma_{CP} \right] \cdot b_w \cdot d \qquad (1)$$

The toughness parameter ($f_{eq(0.6-3)}$ in the present work) can be included, as above discussed, so that the following relationship can be written:

$$V_{Rd,ct,FIBERS} = \left[\begin{array}{l} \frac{0.18}{\gamma_c} \cdot k \cdot (100 \cdot \rho_1 \cdot (1 + 2.5 \cdot \frac{f_{eq(0.6-3)}}{0.30 \cdot (f_{ck})^{2/3}}) \cdot f_{ck})^{1/3} + \\ +0.15 \cdot \sigma_{CP} \end{array} \right] \cdot b_w \cdot d \qquad (2)$$

where, besides all coefficients defined in EC2 (2003), $0.30(f_{ck})^{2/3} = f_{ct}$, where f_{ct} is the tensile strength of concrete.

Since parameter $f_{eq(0.6-3)}$ usually ranges between 1 and 5 MPa, the increase in shear strength due to fiber reinforcement can become twice as much as that of the member without fibers. It can be concluded that the formulation proposed give reasonable values for most practical applications, and confirm the experimental results carried out at the University of Brescia. A comparison between the proposed formulation against a wide set of experimental results obtained within a Brite-Euram program (2002) was quite satisfactory (Minelli, 2005).

4 MINIMUM SHEAR REINFORCEMENT

The primary role of the minimum shear reinforcement is to limit the growth of inclined cracks, to improve ductility, and to ensure that the concrete contribution to shear resistance is maintained at least until yield of the shear reinforcement.

The current codes require the minimum amount of transverse reinforcement, if necessary, to be able to resist a portion of the shear force. The current EC2 states that the minimum transverse reinforcement should be designed as follows:

$$V_s = \frac{A_w \cdot f_{yk} \cdot d}{s} \geq 0.08 \cdot \sqrt{f_{ck}} \cdot b_w \cdot d \qquad (3)$$

By arranging the above equation, one can find out the minimum transverse reinforcement percentage:

$$\rho_{w,\min} = \frac{0.08 \cdot \sqrt{f_{ck}}}{f_{yk}} \qquad (4)$$

The same approach can be followed for FRC, by imposing that the minimum amount of shear reinforcement be carried by fibers only. The minimum FRC toughness to satisfy minimum shear resistance can be determined by imposing that:

$$V_{Rd,ct,FIBERS} - V_{Rd,ct} \geq 0.08 \cdot \sqrt{f_{ck}} \cdot b_w \cdot d \qquad (5)$$

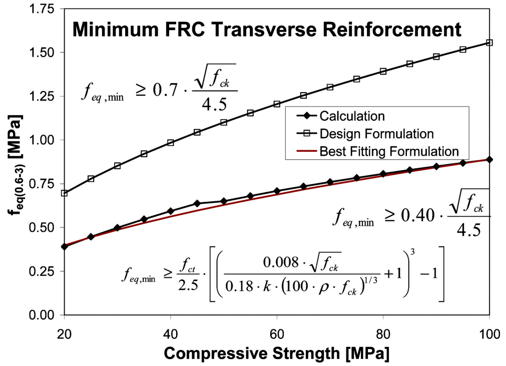

Figure 5. Minimum transverse reinforcement, in terms of fibers, calculated and required.

By rearranging the above equation, one can calculate the minimum value of the post-cracking strength of the FRC composite that provides the minimum shear resistance, as required by EC2.

$$f_{eq,\min} \geq \frac{f_{ct}}{2.5} \cdot \left[\left(\frac{0.008 \cdot \sqrt{f_{ck}}}{0.18 \cdot k \cdot (100 \cdot \rho \cdot f_{ck})^{1/3}} + 1 \right)^3 - 1 \right] \tag{6}$$

The last equation, in which all coefficients refer to EC2 (2003), is a function of the main parameters influencing the shear response of a member. However, it may be difficult to transfer Equation (8) into a simple design procedure. By considering the size effect factor k = 1.2 (equivalent to an effective depth of the member d of 5000 mm) and a reinforcement ratio equal to the minimum longitudinal reinforcement required by EC2, it is possible to minimize the shear strength in order to safely maximize the value of $f_{eq(0,6-3)}$ to be provided to the matrix as the minimum transverse reinforcement requirement. In doing so, an easier expression can be found as follows:

$$f_{eq,\min} \geq 0.4 \cdot \frac{\sqrt{f_{ck}}}{4.5} \tag{7}$$

The last two equations are plotted in Figure 5, that shows that the rigorous conservative expression in Equation (8) and the best fitting formulation of Equation (9) give essentially identical results.

From a design point of view, considering that the current codes give a limitation also to the maximum allowable spacing of transverse reinforcement, the design value of the residual strength, also plotted in Figure 5, should meet the following requirement, in which a partial safety factor was included:

$$f_{eq,\min} \geq 0.7 \cdot \frac{\sqrt{f_{ck}}}{4.5} \tag{8}$$

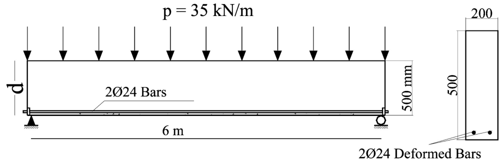

Figure 6. Beam geometry.

5 DESIGN EXAMPLE

The shear design of a beam, by using either regular transverse reinforcement or steel fibers will be presented in the following.

Figure 6 shows the geometry, the loading and the longitudinal reinforcement detail of the beam, which is supposed to be subjected to a load of 35 kN/m under ULS conditions. An effective depth of 460 mm, a characteristic cylinder concrete strength of 30 MPa under compression and of 2 MPa under tension, and a characteristic value of the yield strength of the steel equal to 435 MPa are assumed.

When designing the conventional transverse reinforcement, the concrete contribution to shear strength can be calculated as follows (EC2, 2003):

$$V_{Rd,ct} = \left[0.12 \cdot k \cdot (100 \cdot \rho_1 \cdot f_{ck})^{1/3} \right] \cdot b_w \cdot d = 56\,kN$$

from which the minimum amount of transverse reinforcement (2 stirrups ϕ6@300 mm) spans 3.2 m in the central part of the beam while 2 stirrups ϕ8@300 mm are placed in the support regions, where a shear reinforcement is required (Figure 7).

By adopting a FRC with a post-cracking strength $f_{eq(0.6-3)} = 2.7$ MPa (obtained with 30 kg/m^3 of steel fibers), the FRC shear strength is derived as follows:

$$V_{Rd,ct,FIBERS} = \left[0.12 \cdot k \cdot \left(100 \cdot \rho_1 \cdot \left(1 + 2.5 \cdot \frac{f_{eq(0.6-3)}}{0.3 \cdot (f_{ck})^{2/3}}\right) \cdot f_{ck}\right)^{1/3} \right] \cdot b_w \cdot d = 92\,kN$$

The region requiring the minimum amount of transverse reinforcement spans 5.2 m and only 0.8 m of the beam, around the support regions, requires stirrups. One should note that the minimum transverse reinforcement requirement is totally satisfied by the fibers themselves. In fact:

$$f_{eq,\min} \geq 0.7 \frac{\sqrt{f_{ck}}}{4.5} = 0.7 \frac{\sqrt{30}}{4.5} = 0{,}85\,MPa \leq 2.7\,MPa$$

Two legged stirrups ϕ6@300 mm are placed in the region requiring conventional shear reinforcement.

Figure 7 shows the transverse reinforcement layout in the two cases discussed herein. The use of FRC reduces the portion of the beam requiring stirrups for equilibrium and increases the beam portion where the shear action is lower than the shear strength of FRC.

Such an interesting outcome is therefore very promising with respect to a number of structural

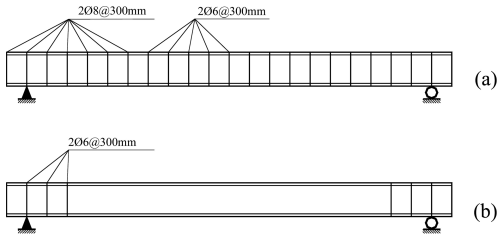

Figure 7. Measured prestressing forces in the bottom strands.

applications, one above all those related to the prefabrication industry, for which handling and properly placing of the transverse reinforcement could represent a significant extra-cost especially while dealing with complicated structural cross-sections.

6 CONCLUDING REMARKS

A survey on recent shear tests on deep beams with little or no shear reinforcement as well as a design equation for shear strength of FRC beams (without conventional reinforcement) are discussed in this paper. An equation for shear strength of FRC without transverse reinforcement is proposed. It includes the shear contribution of fibers as an enhancement of the concrete contribution by modifying the longitudinal reinforcement ratio considered by EC2. As it increases, the longitudinal reinforcement limits the growth of shear-critical crack allowing a greater transfer of stresses (whether tensile or shear).

The proposed equation is based on FRC performance (residual post-cracking strength) which is the more significant index for FRC structural design. It can be easily applied and transferred into practice. FRC performance can be measured by performing simple fracture mechanics tests (on small beams). Many laboratories already have the necessary equipment for performing these material tests.

Further key-points were discussed in this paper as herein recalled:

- Fibers can be considered as reinforcement spread out all over the depth of a member. With this assumption, they act in determining a crack pattern more distributed and complex, avoiding or certainly delaying the localization of a major shear-critical crack responsible of the early brittle collapse.
- The minimum transverse reinforcement requirement can be met by using fibers, in sufficient amount and with minimum performance in terms of toughness.

A simple practical design application was also presented.

Further development of the model, especially with reference to the size effect issue, are expected soon.

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