Investigation of cracks surface roughness and shear transfer strength of cracked HSC

V. P. Mitrofanov
Poltava National Technical University, Poltava, Ukraine

ABSTRACT: The definition of roughness characteristic of rupture crack surface (RCCS) is introduced. The results of RCCS investigation of different strength concretes on the special experimental installation are stated. The RCCS differences of normal and high strength concretes are displayed. On the RCCS basis the criterion of existence of interlock (CEI) of roughness on the crack sides is introduced. The model of resistance to shear in crack is worked out.

1 INTRODUCTION

The problem of the resistance to shear in crack of plain and reinforced concrete (RC) elements is significantly important for RC structures. This problem concerns the inclined cracks of RC beams and columns under shear force action (Fenwick & Paulay 1968, Taylor 1970), the elements loaded by axial tension and shear (rectangular containers and silos) and other cases (Walraven & Stroband 1999). But the above mentioned issue is not yet solved with the sufficient profundity. So, all known design methods (Mattock et al. 1976) are mainly empirical, i.e. they correspond to the lowest level of this problem solution and hide the physical reality. That is why it is necessary to develop physically grounded and clear models for the strength design of cracks in plain and RC elements subjected to shear.

2 PRINCIPAL NOTIONS

Depending on character of roughness on the crack surface and mutual movement of the crack sides the roughness interlock in crack may occur in certain cases and the one may be absent in some other ones. Therefore it is necessary that the solution of the strength problem of interlock in crack should begin with the wording criterion of existence of interlock (CEI) of roughness on the crack sides. The latter was suggested in (Mitrofanov 1982), where the concept of roughness characteristic of rupture crack surface was introduced as a basis.

The roughness characteristic of the rupture crack surface (RCCS) is considered as the trajectory equation

$$\delta = f(w)$$  \hspace{1cm} (1)

of the mutual progressive movement of crack sides (beginning from the dense closed state of crack sides) that take place, when corresponding one to other juts and hollows of roughness have the contact without the deformations. In (1) $\delta$, $w$ = tangential and normal mutual displacements of crack sides. From (1) the CEI follows

$$\delta_1 \geq f(w_1), \quad w_1 < l_m,$$  \hspace{1cm} (2)

where $\delta_1$, $w_1$ = tangential and normal mutual displacements of crack sides under the real movement of concrete structure, $l_m$ = the most height of roughness juts.

3 EXPERIMENTAL INVESTIGATION

The RCCS was studied on the special testing installation (Fig. 1), using the concrete cubic specimens, which were previously split in two equal halves I and II by the rupture crack. The test cubics had the rib length of 10, 15 and 20 cm. The normal strength concretes $R_b = 25 \ldots 45$ MPa and the high strength concrete $R_b = 100$ MPa were used.

Before the test, the specimen halves were densely closed. The half I was immovably attached between the rests 1. The half II was attached to mobile strips 7 and 8 that can transfer to half II the displacements $w$ and $\delta$ (Fig. 1).

In order to obtain the test point of trajectory (1), at first instance, the mobile specimen half II received little displacement $w$ and then displacement $\delta$ until the interlock moment of crack roughness. Such measuring cycles of gradually increasing values $w$ and $\delta$ were repeated several times until reaching the great value $w = 5 \ldots 6$ mm when the roughness interlock became
impossible. Then the specimen halves I and II were again densely closed and the above described measurings were repeated 3–4 times per specimen.

The test points multitude allowed to approximate the curve (1) by the parabola (Fig. 2)

\[ \delta = pw^2 + qw \]  

(3)

where for the normal strength concretes \( R_b = 25 \ldots 45 \) MPa \( p = 0.1875 \) mm\(^{-1} \), \( q = 0.175 \), \( l_m = 5 \) mm, for the high strength concrete \( R_b = 100 \) MPa \( p = 0.281 \) mm\(^{-1} \), \( q = 0.262 \), \( l_m = 4 \) mm. Thus, the concretes of normal and high strength distinguish by the parameters \( p \) and \( q \) of the RCCS (3), i.e. the RCCS takes into account the influence of the concrete type and the strength on geometrical properties of roughness on the rupture crack surface.

If the roughness is imagined as the multitude of corresponding one to other right wedge-shaped juts and hollows, then according to (3), the lower juts are more abrupt and the higher ones are more gentle (Fig. 3).

4 DESIGN MODEL

The RCCS notion allows to ground the crack roughness characteristic method (CRCM) for the strength design of shear transfer in concrete cracks. The main concepts of the CRCM are the next ones.

4.1 Geometrical model of roughness

On adjacent parallel plane and smooth sides of the crack the congruous juts and hollows of roughness are placed. The roughness juts are assumed as plane with the isosceles-triangular contour and middle plane parallel to direction of the mutual crack sides shear \( \delta \). The model roughness juts are characterized by the variable angle \( 2\alpha \) near the top, length \( l \) and the same width
The roughness model together with the RCCS (3) leads to the relationship between parameters $\alpha$ and $l$ of juts

$$\tan \alpha = p l + q$$  \hspace{1cm} (4)

and for the movement case (2) the one allows to determine the ultimate height of interacting (bearing) roughness juts

$$l_{bc} = (\tan \varphi - q) / p \leq l_m,$$  \hspace{1cm} (5)

where $\varphi = \angle$ between displacement vector $r(w, \delta)$ and normal $v$ to crack (Fig. 3). In consequence the multitude of roughness juts depending on contribution to resistance to shear are divided into three groups, which correspond to three parts on the RCCS (Fig. 4):

1. $0 < w < l_f$ – failed juts,
2. $l_f < w < l_{bc}$ – bearing juts,
3. $l_{bc} < w < l_m$ – non-interacting juts (the last ones are absent when $l_{bc} = l_m$).

4.3 Strength of separate roughness jut

It is considered that the model roughness jut is loaded on the inclined part $O C$ by the normal $\sigma_n$ and tangential $\tau_n$ stresses which are reduced to the resultant $R$ with components $T$ and $N$ (Fig. 5) in the direction of axes $t$ and $\nu$ (Fig. 3). In accordance with (Mitrofanov 1990) the following ultimate forces are obtained

$$T = m \xi(x, y, \alpha, B) \tan \alpha b l_{bc},$$  \hspace{1cm} (7)

$$N = T \eta,$$  \hspace{1cm} (8)

where

$$m = R_b - R_{bt}, \quad B^2 = \left[1 + \frac{1}{3} \frac{X(1-X)^2}{X} \right], \quad \lambda = \frac{R_{ht}}{R_b},$$  \hspace{1cm} (9)

$$\xi = \frac{2B}{(1 + \frac{1}{4} \frac{X(1-X)^2}{X})^{-1}} \frac{xy + 1}{X} \cdot \frac{2 \cdot \tan \alpha}{X},$$  \hspace{1cm} (10)

$$x = v_x / v_y, \quad y = \tan(\alpha + \gamma),$$  \hspace{1cm} (11)

$R_b, R_{bt}$ = concrete strength under the axial compression and tension, $v_x, v_y$ = velocity components of the
mobile part $I$ of concrete juts in failure state, $\gamma = \text{angle (Fig. 5)},$

\begin{align*}
L_{bc} &= l - w, \\
\eta &= c \tan \alpha
\end{align*}

(12) \quad (13)

4.4 Ultimate stresses transferred in crack

The ultimate tangential $\tau_{\text{int}}$ and normal $\sigma_{\text{int}}$ stresses of the interlock in crack are determined as the sum of corresponding components $T$ and $N$ of the multitude of bearing roughness juts placed on the unit area of crack surface

\begin{align*}
\tau_{\text{int}} &= \frac{n}{\sum_{l_j} T(1) f(l) dl} = nT\bar{T} e^{-\lambda l}, \\
\sigma_{\text{int}} &= \tau_{\text{int}} \eta;
\end{align*}

(14) \quad (15)

where it is assumed

$$\bar{T} = (w + l_{bc}) / 2, \alpha = \pi / 4,$$

$$\lambda = 1 / \bar{T}, \quad l_j = w.$$

(16) \quad (17)

Taking into account (7)–(13), (3), (5), from (14)–(17) the design formulas may be obtained as follows

$$\tau_{\text{int}} / m = K \frac{\bar{T} - w}{l_m} \tau_{\text{int}} \eta,$$

$$\sigma_{\text{int}} = \tau_{\text{int}} \eta,$$

(18) \quad (19)

where

$$K = bl_m n,$$

$$\omega = \frac{x(y, \alpha, B) e^{-w/\bar{T}}}{\eta} = 1,$$

$$x = w / \delta, \quad y = l_{bc} / w,$$

(20) \quad (21) \quad (22)

$\xi$ is determined from (10) using (22), (16), (9).

On basis of the experiments (Fenwick & Paulay 1968, Taylor 1970, Walraven & Stroband 1999) and relationship (18) the reduced density of roughness juts $K$ (20) was determined and the one turned out to be the function of the displacements ratio $\delta / w$

$$K = 2 C (\delta / w) / \left[3 + (\delta / w)\right],$$

(23)

where the factor $C$ takes into account the cube strength $R(MPa)$ influence on the reduced density of roughness juts

$$C = 353.6 / \left(339.56 - R + 0.1074 R^2 \right)$$

(24)

5 CONCLUSIONS

The obtained relationships show that the shear transfer strength in the concrete crack is conditioned by two factor groups: (1) geometrical ($p, q, l_m, K$) and strength ($R_h, R_{bh}$) parameters of the crack surface roughness, (2) mutual displacements of crack sides ($\delta, w, \tau_{\text{int}} \eta = \delta / w$). The formula (18) with the above mentioned $K$ has led to the variation coefficient of deviation from 1 ratio $\tau_{\text{test}} / \tau_{\text{calc}} \eta = 12\%$.

The systematic investigations of the RCCS concerning the wide range of concrete types and strength are required. More then that, the likely direction of the model improvement is the consideration of the roughness juts parameters $l$ and $\alpha$ as discrete stochastic values and the use of corresponding suitable law of distribution density of roughness juts number.

The reinforcement contribution into resistance to shear in concrete cracks may be taken into account according to (Mitrofanov 1982).

REFERENCES


